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STABILITY OF THERMOCAPILLARY CONVECTION AND REGIMES OF A FLUID FLOW ACTED UPON BY A STANDING SURFACE WAVE

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It has been established that, in the case where a standing surface wave acts on a thermocapillary-convection flow in a cylindrical volume, there arises an oscillating-convection zone between the laminar and turbulent regimes of flow. It is shown that the boundary between these regimes is determined by the amplitude δ and the number of periods n of the standing wave and is practically independent of the Marangoni number and the oscillation frequency of this wave. At n = 2, in the range 0.004 < δ < 0.006, the parameters of the fluid cease to oscillate. The mechanisms by which the thermocapillary convection in closed volumes loses its stability are discussed.

Introduction. The great interest shown in thermocapillary convection in recent years is explained by the fact that this convection is of importance for a number of engineering applications, first of all for crystal growing by the method of zone melting free of a crucible process (floating-zone method) under the microgravity conditions aboard a space vehicle. For optimization of a technological process, it is necessary to select an appropriate regime of a fluid flow, in particular, to eliminate oscillations of the fluid parameters or decrease them to a minimum. Because of this the study of the stability and regimes of thermocapillary flows is of importance not only for theoretical hydromechanics but also for technical applications.

The study of the stability of thermocapillary-convection fluid flows by the linear method, based on the use of infinitely small disturbances and on the linearlization of the equations of motion for an infinitely large free fluid surface, was begun almost 50 years ago [1, 2]. In more recent works [3–6], the linear theory of flow stability was used for solving some particular problems. In [6], the deformation of the free surface of a fluid was estimated for a definite capillary number K_σ. Another "energy" method of investigating the stability of fluid flows is based on the use of the equation for the rate of change in the energy of a disturbance in a fluid volume [7]. The application of the variation principle to the integral Euler–Lagrange equations obtained from the volume energy integrals allows one to determine the maximum value of the Marangoni number, below of which a fluid flow is stable [8–10]. In [9], the deformation of the free surface of a fluid was estimated on the condition that the parameter $\mathrm{Pr}/K_\sigma \neq 0$, i.e., K_σ is finite. In [10], the indicated integrals were calculated using the temperature and velocity fields in a convective cell, calculated for the Marangoni number characteristic of a very unstable laminar flow (Ma = 100). The calculation data obtained in [10] are in good agreement with the experimental data of [11]. The calculations carried out in [12] has shown that the free surface of a laminar fluid flow of near-boundary stability experiences a small static deformation under the action of a kinetic head. As surface experiments performed in [13] have shown, only in the oscillation-convection regime does the oscillation amplitude of the free surface of a small silicone-oil column reach approximately 1% of its radius and the frequency of the temperature oscillations equal to the frequency of oscillations of a point on the free surface of the column.

Radically new opportunities for investigation of the stability of a fluid flow with a free surface open up when the nonlinear Navier–Stokes equations are solved on the condition that the fluid free surface oscillates as a standing wave under the action of a vibration. In this case, not only the disturbance parameters, having no influence on the flow stability, but also the mechanisms of formation of an oscillation convection can be determined. The present investigation was carried out with the parameter ξ_c used in [14, 15] for analysis of the segregation of an impurity and showed a high sensitivity to a change in the structure of a flow.

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Formulation of the Problem and Computational Procedure. A flow of an incompressible viscous fluid in a cylinder of length L and radius R is considered. A standing wave with an amplitude b and a number of periods n acts on the free surface of this cylinder. The cylinder surface is acted upon by an external heat flow, and the temperature of the solid face surfaces bounding the fluid is equal to T_0. At the face surface, $z = 0$ (crystallization boundary) and an impurity is precipitated, and the other face surface $z = j$ and the side surface are impenetrable for the impurity.

The dimensionless system of two-dimensional Navier–Stokes equations of heat and mass transfer in the Boussinesq approximation has the form

$$\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} = -\frac{\partial p}{\partial r} + \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right), \tag{1}$$

$$\frac{\partial v}{\partial \tau} + u\frac{\partial v}{\partial r} + v\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2}\right), \tag{2}$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{\partial z} = 0\,, \tag{3}$$

$$\frac{\partial \theta}{\partial \tau} + u\frac{\partial \theta}{\partial r} + v\frac{\partial \theta}{\partial z} = \mathrm{Pr}^{-1}\left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial z^2}\right), \tag{4}$$

$$\frac{\partial C}{\partial \tau} + u\frac{\partial C}{\partial r} + v\frac{\partial C}{\partial z} = \mathrm{Sc}^{-1}\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2}\right). \tag{5}$$

The boundary conditions are as follows:

$$z = 0: \quad u = v = 0\,, \quad \theta = 0\,, \quad -\frac{\partial C}{\partial z} = \mathrm{Re}_{\mathrm{cr}}\mathrm{Sc}\,(1 - k_0 C_{\mathrm{s}})\,, \tag{6}$$

$$z = 1: \quad u = v = 0\,, \quad \theta = 0\,, \quad \frac{\partial C}{\partial z} = 0\,, \tag{7}$$

$$r = 0: \quad \frac{\partial u}{\partial r} = \frac{\partial v}{\partial r} = \frac{\partial \theta}{\partial r} = \frac{\partial C}{\partial r} = 0\,, \tag{8}$$

on the free side surface oscillating by the standing-wave law $r = \xi(z) = b\,\sin\,(2\pi zn/L)\,\cos\,\Omega\tau$,

$$\frac{\partial \theta}{\partial r} = -A\,(\cos\alpha)\,\exp\,[-B\,(z - L/2)^2]\,, \quad \frac{\partial C}{\partial z} = 0\,, \quad u = b\,\sin\,(2\pi zn/L)\,\sin\,\Omega\tau\,, \tag{9}$$

$$\frac{\partial v}{\partial r} = -\mathrm{MaPr}^{-1}\frac{\partial \theta}{\partial z}\,. \tag{10}$$

Here, (8) is a symmetry condition and (10) defines the thermocapillary effect.

System (1)–(10) was numerically solved by the finite-difference method. The difference grid along the radius was constructed for a standing wave with an oscillation amplitude $(R + b)$. In the process of calculations, the boundary

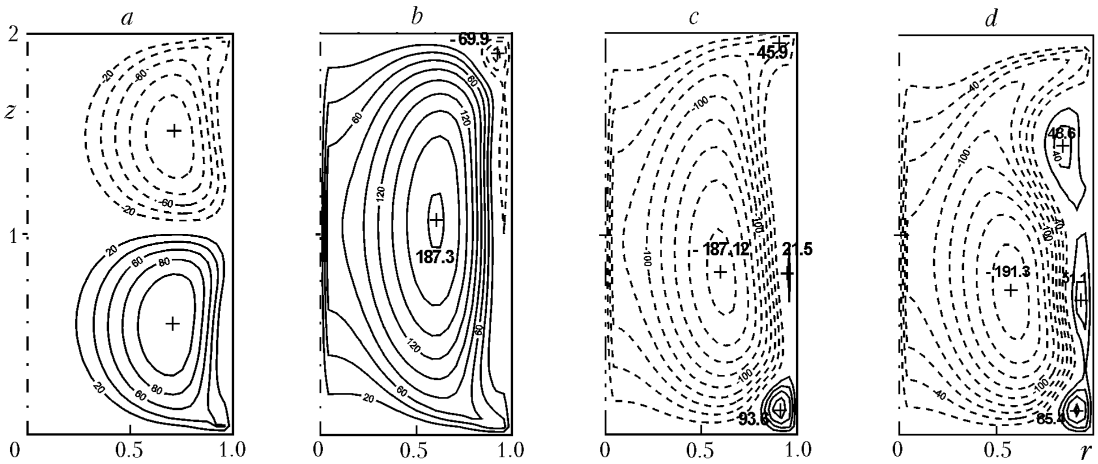


Fig. 1. Fields of the flow function (the solid lines denote the clockwise flow, the dashed lines denote the anticlockwise flow): a) Ma/Pr = $6.6 \cdot 10^4$; b) Ma/Pr = $6.6 \cdot 10^4$, Ω = 2500, δ = 0.0043; c) $8.6 \cdot 10^4$, 2500, 0.004; d) $6.6 \cdot 10^4$, $7.8 \cdot 10^4$, 0.004.

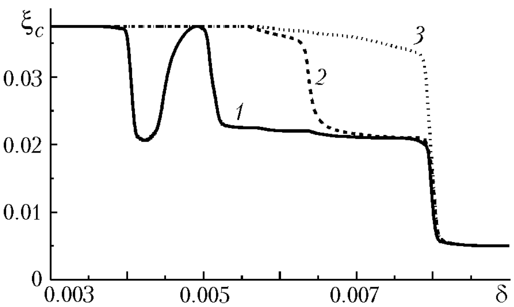
points on the oscillating free surface were determined by a definite law of motion. It is assumed that a point lies on the fluid boundary if its r coordinate exceeds half the grid pitch. An implicit finite-difference spatial scheme of the third order of accuracy is constructed on a five-point pattern. The Neumann problem for the pressure increment is solved using a computational algorithm and velocity values at half-integer points. This provides the fulfillment of the difference analog of the incompressibility condition. The convergence and stability of the solution is provided at the following conditions for the space and time steps:

$$h_{\min} \le \frac{0.3}{U_{\max}^{1/2}} \quad \text{and} \quad \Delta\tau \le \frac{2}{U_{\max}} . \tag{11}$$

In the calculations, the time step is corrected automatically for the fulfillment of the second condition (11). The pressure increment is determined by the conjugate-gradient method, and the temperature, velocity, and concentration are determined by the implicit Lanczos method. The stability of the difference scheme and the convergence of the solution of the problem are verified by comparison of the data obtained with the available calculation data obtained as a result of solution of the test problem on a fluid flow in a cavity with a moving cover, including the case of a three-dimensional flow, and experimental data [16].

Results of Calculations. The problem formulated was solved for a cylindrical fluid volume of elongation L/R = 2, the free surface of which with parameters A = 0.8 and B = 0.3 is acted upon by a heat flow. The main calculations were carried out for fluids with Pr $\le$ 1 (0.018 and 0.023), characteristic of the majority of semiconductor materials, and the other calculations were carried out for Pr = 1. The data obtained have shown that the Prandtl number is a determining parameter only in combination with the Marangoni number of the form of Ma/Pr. This parameter characterizes the intensity and structure of the initial thermocapillary flow that is not acted upon by a standing surface wave (SSW).

The procedure of investigating the stability of a fluid flow was as follows. At first, calculations were performed for the process of formation of a stationary flow without an SSW action, and then the action of a standing wave with oscillations of definite amplitude and frequency were taken into account. The calculations were carried out as long as a new stationary or oscillation regime with steady-state values of the amplitude and frequency of oscillations of the fluid parameters was established. A new regime called for from one hundred to several thousands of SSW oscillation periods to establish, and the number of these periods increased with increase in Ω. The ranges of change in the problem parameters being investigated were as follows: δ = 0.002–0.012, Ω = 440–$7.8 \cdot 10^4$, n = 1–15, Ma/Pr = $6 \cdot 10^3$–$8.6 \cdot 10^4$.



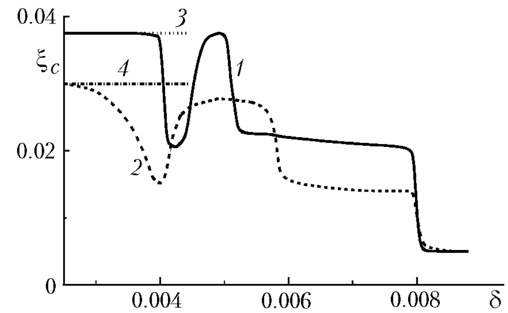


Fig. 2. Dependence of the parameter ξ_c on the amplitude δ of an SSW for three values of the SSW oscillation frequencies Ω: Ω = 2500 (1), 1000 (2), and 440 (3). Ma/Pr = $3.9 \cdot 10^4$.

Fig. 3. Dependence of the parameter ξ_c on the SSW amplitude δ for Ma/Pr = $3.9 \cdot 10^4$ (1, 3) and $6.6 \cdot 10^4$ (2, 4) [3) and 4) in the absence of the SSW]. Ω = 2500, n = 2.

The initial structure of a thermocapillary flow with no an SSW action (Fig. 1a) is symmetric relative to the symmetry plane of a definite heat flow ($z = L/2$). Under the action of a SSW, the flow structure changes and takes one of the forms shown in Fig. 1b–d; the flows with these structures are opposite in direction and are obtained from each other by rotation about the axis $z = 1$. In these flows, the maximum values of the flow function have different sings. It should be noted that the flow with a positive flow-function maximum (Fig. 1b) can be considered as conditionally stable. In this case, the position of the convective cells remains unchanged and there occur a small jitter of the streamlines and weak oscillations of the velocity components at the free boundary of the fluid with an applied frequency Ω. The second structure of the flow (Fig. 1c and d) is characterized by a larger dynamics of change. The fluid parameters oscillate with frequencies lower than the applied frequency Ω. In this regime, the frequencies of oscillations of the fluid parameters have values characteristic of a usual oscillating thermocapillary-convection flow (without an SSW action). To determine the mechanism of appearance of different flow structures in the case of action of an SSW on a thermocapillary-convection flow, it is necessary to consider the ratio between the oscillation frequency of the SSW Ω and the eigenfrequency of a convective cell Ω_f considered in [17]. When Ω is much larger than Ω_f, the flow structure formed under the action of the SSW is practically independent of Ω if the other wave parameters are identical. This is apparent when the structures shown in Fig. 1c and d are compared. Moreover, the formation of a new structure depends insignificantly on the intensity of the initial thermocapillary flow and on the value of the parameter Ma/Pr. Even though Fig. 1 shows the flow structures obtained in the case where two SSW periods (n = 2) are within the free surface, similar flow patterns are also formed at other values of n.

If the frequencies Ω and Ω_f are commensurable (Fig. 2), there exists an explicit dependence of the flow stability on the ratio between these frequencies. For a thermocapillary flow arising at Ma/Pr = $3.9 \cdot 10^4$, the eigenfrequency of a convective cell Ω_f = 480 [17]. If a standing wave oscillates with a frequency Ω = 440, there takes place an approximate equality between Ω and Ω_f (curve 3) and the oscillations of the SSW correspond to the integral period of the fluid rotation in the convective cell. In this case, the flow is stable in a wide range of change in the SSW amplitude as long as $\delta \approx 0.008$, which corresponds to the break of the flow stability and the change to the turbulent regime. If the frequency Ω is much higher than Ω_f (curve 1, Ω = 2500), the indicated frequency matching is not realized and, when the SSW amplitude changes, there takes place a complex nonlinear interaction of a disturbance with the thermocapillary flow, with the result that different flow regimes arise: from weakly oscillating flows (illustrated in Fig. 1b–d) to conditionally stable flows. In this case, an unusual phenomenon is the appearance of a conditionally stable flow in the region of the oscillating flow, the indication of which is a local maximum of ξ_c at δ = 0.0044–0.0051 (specific "resonance" effect). Curve 2 corresponds to the intermediate case where the frequency of the applied disturbance (Ω = 1000) is approximately two times higher than the eigenfrequency of the convective cell. The indicated "resonance" effect arises when the structure of the initial thermocapillary-convection flow, which is not acted upon by an SSW (see Fig. 1a), and the SSW configuration with n = 2 are mutually symmetric.

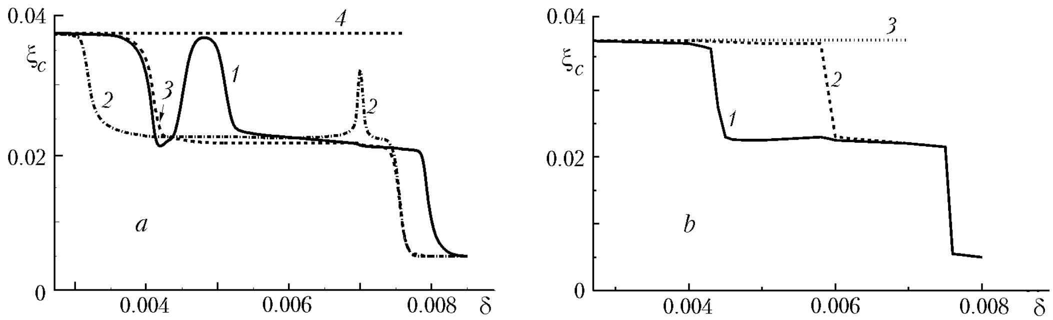
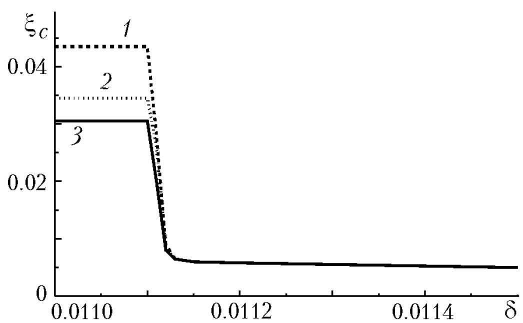


Fig. 4. Dependence of the parameter ξ_c on the SSW amplitude δ for even values of n (a): n = 2 (1), 4 (2), and 6 (3) [4) in the absence of the SSW], and for odd values of n (b): n = 5 (1) and 3 (2), [3) in the absence of the SSW]. Ma/Pr = $3.9 \cdot 10^4$, Ω = 2500.



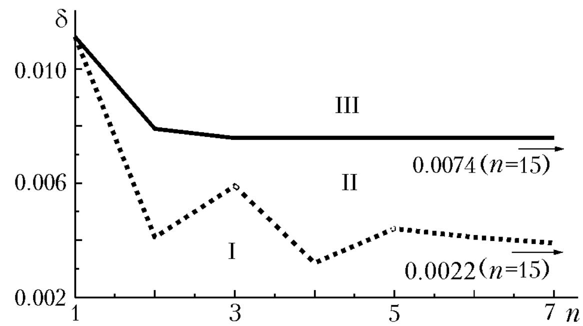


Fig. 5. Dependence of the parameter ξ_c on the SSW amplitude δ at n = 1: Ma/Pr = $2.19 \cdot 10^4$ (1), $5.5 \cdot 10^4$ (2), and $6.6 \cdot 10^4$ (3). Ω = 2500.

Fig. 6. Boundaries between the regimes of flow [I) laminar flow, II) oscillating or weakly disturbed flow, III) turbulent flow) dependeng on the number of SSW oscillation periods n. Ma/Pr = $3.9 \cdot 10^4$, Ω = 2500.

The data presented in Figs. 2 (curve 1) and 3 show that, at $\Omega > \Omega_f$ and n = 2, the change from the conditionally stable flow to a weakly oscillating flow happens at δ = 0.003–0.004 and that this change and the width of the transition region are determined by the parameter Ma/Pr. The regime of weak oscillations changes for the turbulent regime of flow abruptly at $\delta \approx 0.008$, and this change is independent of the oscillation frequency of the SSW and the intensity of the initial thermocapillary flow.

Below are figures illustrating the influence of the mutual symmetry of the initial thermocapillary flow and the SSW configuration on the character of the fluid flow. Figure 4a shows the dependence $\xi_c(\delta)$ determined for one and the same values of the thermocapillary-flow-intensity (Ma/Pr = $3.9 \cdot 10^4$) and the SSW frequency (Ω = 2500) and three even values of n: 2, 4, and 6. It is clearly seen that the "resonance" effect, arising as a result of the mutual symmetry of the standing wave and the initial structure of the thermocapillary flow, degenerates at n = 4 into a small peak and disappears at $n > 4$. If the number of the SSW periods is odd and the above-indicated symmetry is absent, the "resonance" effect does not appear (Fig. 4b). In this case, the zone of weak oscillations is narrower at n = 3 since the mutual asymmetry of the SSW and the initial flow is more evident in this case than at n = 5. The mutual asymmetry of the initial flow and the disturbance is even stronger at n = 1 (Fig. 5). In this case, the zone of weak oscillations is absent and the stable flow is transformed abruptly into a turbulent flow.

In all the above-described examples, the value of the parameter ξ_c depends critically on the character of the fluid flow. In all the cases where the value of this parameter remains identical to that in the absence of a standing wave (including the "resonance" region at n = 2), a conditionally stable flow with a structure shown in Fig. 1b is realized. In all the cases where ξ_c decreases markedly (the minima on the curves $\xi_c(\delta)$ for n = 2 and the horizontal lines

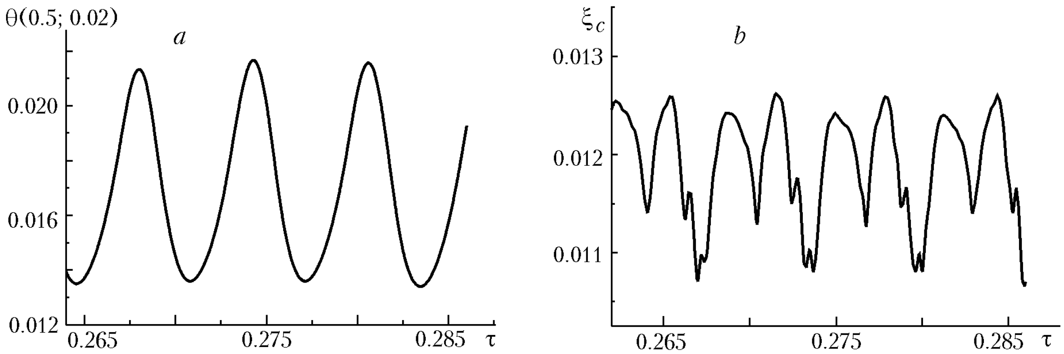


Fig. 7. Oscillations of the temperature (a) and concentration parameter ξ_c (b) of the fluid acted upon by an SSW.

for the standing waves with $n > 2$), an oscillating flow with a structure similar to the structures shown in Fig. 1c and d arises. In this regime, the dimensional oscillation frequencies of the fluid parameters, excepting the velocity components at the oscillating free surface, fall within a narrow range (from 100 to 200) corresponding to the eigenfrequencies of the convective cells formed [17].

Figure 6 shows the dependence of the amplitude of an SSW δ on the number n at the boundaries between the three regimes of fluid flow: the lower curve separates the regions of the conditionally stable flow (I) and the flow with small oscillations of the fluid parameters (II), and the upper curve separates the oscillation (II) and turbulent (III) regimes. It is seen that, at $n > 6$, the value of δ at the upper boundary remains practically unchanged when n increases, and the value of δ at the lower boundary decreases slowly with increase in n. All the fluid flows arising at $n > 6$ can be considered as flows with a deformed free surface of the type of a ripple. At $n = 15$, $\delta = 0.0022$ at the lower boundary and $\delta = 0.0074$ at the upper boundary; these values can be considered as asymptotes at any other larger values of n. The lower boundary was determined on the condition that the oscillation amplitude of the parameter ξ_c reaches 0.1%. The temperature oscillations at the point selected ($r = 0.5$, $z = 0.02$) approached 1% in this case. Above the upper boundary, the oscillation amplitudes of all the fluid parameters comprised several tens of percent and the spectrum represented a set of frequencies. Even though the data presented in Fig. 6 were obtained for concrete values of Ma/Pr and Ω, they can serve for orientation in the case where the indicated parameters take other values because, as was shown above, the characteristics of the fluid-flow regimes change insignificantly with change in these parameters.

The regime of weak oscillations is characterized by a small number of oscillation frequencies (two-tree frequencies). In this case, for the parameter ξ_c, a low frequency, close to the frequency characteristic of the usual oscillation thermocapillary-convection regime (F = 100–200), manifests itself most markedly. This frequency increases somewhat with increase in the wave amplitude δ. The oscillation frequency of an applied SSW Ω is also present in the spectra of oscillations of the velocity components; this is especially true for the points at the free surface of the fluid. This frequency is absent in the spectrum of the parameter ξ_c at large values of Ω. Figure 7 shows, as an example, the temperature oscillations at the point with the above-indicated coordinates and the oscillations of the parameter ξ_c for the flow at $\mathrm{Ma/Pr} = 6.6 \cdot 10^4$, $n = 2$, $\delta = 0.0055$, and $\Omega = 7.8 \cdot 10^4$.

Discussion of the Results Obtained. An arbitrary variation of the determining parameters of the problem in the process of numerical simulation makes it possible to perform investigations in a wide range of dimensional parameters. It has been established that the stability of a thermocapillary-convection acted upon by a harmonic disturbance in the form of an SSW depends on the mutual spatial symmetry of the flow and the disturbance as well as on the ratio between the frequency of the harmonic oscillation and the eigenfrequency of a convective cell. If the first frequency is much larger than the second one, the oscillation frequency of the SSW Ω drops out of the number of determining parameters, which is reasonable because the effect is attained when a harmonic disturbance is repeated many times (several hundreds or thousands of SSW oscillation periods). In this case, the effects observed and the positions of the boundaries separating the regimes of flow are determined only by the amplitude δ and the number of oscillation periods n of the standing wave. It should be noted that the processes responsible for the appearance of a conditionally stable flow ("resonance") in the region of a weakly oscillating flow in the case where the initial flow is mutually symmetric to the SSW configuration (n = 2 and 4) remain as yet unclearly understood and call for additional investigations. An important result is that the dependence of the solution of the problem on the number of SSW oscillation periods degenerates at $n > 6$; in this case, the disturbance on the free surface of the fluid takes the form of a ripple.

The formation of two oppositely directed different-type flows in the case of action of an SSW on a laminar thermocapillary-convection flow (Fig. 1b and c), revealed in the process of numerical calculations, is not something unusual. This phenomenon is known in the theory of branching of solutions of nonlinear differential equations and is called imperfect bifurcation. Such situations were observed not only in the viscous-fluid mechanics, but also in the theory of elastic bendings of rods and plates and in the process of magnetization of ferromagnets in the neighborhood of the Curie point.

A peculiarity of the problem being considered, which is of interest for the theory of stability of flows in bounded fluid volumes, is that two interrelated different-type flows arise in one convective cell: a shear flow near the free boundary of the fluid with a monotonic velocity profile having a maximum at the free surface and a flow near the solid surface with a nonmonotone velocity profile (see Fig. 1a). Under the action of an SSW, the stability of the shear flow is disturbed, while the flow near the solid boundary possesses a sufficient stability margin at definite parameters of the initial thermocapillary flow. The combination of the oppositely different states by the stability criterion leads to the appearance of nonlinear phenomena and different regimes of flow, including conditionally stable and oscillating flows.

In [18], the case where two flows with similar structures were combined was considered. One of them was an ordinary thermocapillary flow having a structure similar to the structure shown in Fig. 1a, and the other was a secondary flow, formed by a rotating magnetic field. The secondary flow was similar in structure to the thermocapillary flow; it acted on the velocity profiles near the free surface and the solid walls. A change in the intensity of the secondary flow changed its action on both velocity profiles near the boundaries of the region, with the result that the boundary between the laminar and oscillating regimes of the total flow changed nonmonotonically with change in the intensity of the components. In [19], it was shown that the point of change from the laminar regime to the oscillation thermocapillary regime of flow can be controlled by changing the distribution of the heat flow supplied to the free surface of the fluid. When a narrower zone is heated, a more stable thermocapillary convection arises due to the formation of a flow with lines more planar relative to the free fluid surface. In this case, the change from the laminar to the oscillation regime of flow happens at a larger value of Ma/Pr. The data obtained allow the conclusion that the thermocapillary convection in closed volumes loses its stability in the case where the balance between the stability of the flow near the solid boundaries and the flow near the free surface is disturbed.

Conclusions. Our numerical investigation of the action of a standing surface wave on a thermocapillary convection allowed us to reveal the following effects unknown earlier:

1) lengthy zones of weakly oscillating flows exist between the laminar and turbulent regimes;

2) a stable-flow zone is formed inside the zone of weak oscillations at n = 2;

3) the mutual symmetry of the initial thermocapillary flow and the SSW configuration influences the structure and characteristics of the fluid flow;

4) the flow structures formed and the boundaries separating the flow regimes are independent of the parameter Ma/Pr and the SSW oscillation frequency Ω in the case where $\Omega > \Omega_f$.

The dependences of the boundaries separating the flow regimes on the amplitude δ and the number of SSW periods n were determined. It was shown that the problem on the stability of the thermocapillary convection in a closed volume with a bounded free surface should be solved only with account for the flow near the solid boundaries. The indicated problem can be reduced to the problem on the stability of a convective cell. This postulate was used earlier for substantiation of the correspondence of the oscillation frequency of the flow parameters in the oscillation-convection regime to the eigenfrequency of a convective cell [17]. The oscillation frequency of a convective cell is also of importance for determining the regimes of flow acted upon by a harmonic disturbance.

The revealed effect of formation of weakly oscillating fluid flows, in which the fluid is intensively mixed, can be used in practice, in particular for decreasing the macrosegregation of an impurity in the process of rotation of crystals by the floating-zone method.

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NOTATION

A, B, dimensionless coefficients in (9); b, amplitude of a standing wave, m; c, concentration of an impurity in a fluid, kg/m^3; $C = c/c_0$, dimensionless concentration of an impurity; D, diffusion coefficient of the impurity in the fluid, m^2/sec; f, frequency, Hz; $F = fR^2/\nu$, dimensionless frequency; h, step of the computational grid on the dimensionless spatial coordinates; $K_\sigma = \sigma R/(\rho\nu^2)$, dimensionless capillary constant; k_0, equilibrium impurity-distribution coefficient; L, height of a liquid column, m; Ma = $(\partial\sigma/\partial T)R\Delta T/(\rho\nu\chi)$, Marangoni number; n, number of periods of a standing wave along the length L; p, dimensionless pressure obtained by division of the dimensionless pressure (Pa) by $R^2/\rho\nu^2$; Pr = ν/χ, Prandtl number; R, radius of the undisturbed liquid column, m; r, z, cylindrical coordinates, m; $\mathrm{Re}_{cr} = v_{cr}R/\nu$, dimensionless rate of crystal growth; Sc = ν/D, Schmidt number; T, temperature, K; t, time, sec; u and v, dimensionless velocities along the r and z axes, respectively, obtained by division of the dimensional velocities by the scale ν/R; U_{max}, characteristic dimensional velocity of a flow; α, angle between the normals to the points on the disturbed free surface and the surface $r = R$; $\delta = b/R$, dimensionless amplitude of the standing wave; $\Delta T = T_{max} - T_0$, characteristic temperature difference in the system, K; $\Delta C_s = C_{s,max} - C_{s,min}$, radial difference of concentrations in the fluid; $\Delta\tau$, time step used in the numerical calculations; ν, kinematic viscosity, m^2/sec; ρ, density, kg/m^3; σ, surface tension, N/m; χ, thermal diffusivity, m^2/sec; ω, circular frequency of standing-wave oscillations, sec^{-1}; $\Omega = \omega R^2\nu^{-1}$, dimensionless circular frequency of standing-wave oscillations; $\xi_c = \Delta C_s/C_s$, dimensionless relative radial difference of the impurity concentrations; $\theta = (T - T_0)/\Delta T$, dimensionless temperature; $\tau = t\nu/R^2$, dimensionless times. Subscripts: 0, initial value; cr, crystallization; f, eigenfrequency of a convective cell; max and min, maximum and minimum values of a parameter; s, at the crystallization boundary.

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